

Baylor Talk, May 2025

Speaker: Michael Taylor

Title: Microlocal analysis on contact manifolds: eigenfunction concentration, Weyl formulas, and noncommutative phase space.

Abstract. On a compact Riemannian manifold M , microlocal Weyl formulas specify the mean expectation of a pseudodifferential observable A acting on an orthonormal basis of $L^2(M)$ consisting of eigenfunctions of the Laplace operator. One derives a formula that averages the principal symbol of A over the cosphere bundle of M , displaying a uniform mean distribution of the eigenfunctions. For subelliptic operators on M , matters are different. The eigenfunctions concentrate (in a mean sense) microlocally on the characteristic set of L .

In case M is a contact manifold and L an associated sublaplacian, the characteristic set is the contact line bundle, and one again obtains an integral formula, this time over the unit sphere intersected with this line bundle. However, this integral apparently loses information compared to the elliptic case. This is because the characteristic set is too thin. We discuss how to “blow up” the characteristic set to see finer details of the microlocal concentration of eigenfunctions. This brings in a noncommutative microlocal analysis, involving a class of pseudodifferential operators with symbols based on the representation theory of the Heisenberg group, and it yields as the blow up a noncommutative phase space, carrying a richer microlocal Weyl formula.