2024 Texas Differential Equations Conference

Baylor University

All talks will take place on the 3rd floor in the Sid Richardson Building

Session A: Room 344

8:45 am – 9:00 am Registration

9:00 am – 9:05 am Welcome

9:05 am – 9:30 am Jameson Graber, Baylor University

On Mean Field Games through the Lens of Conservation Laws

Abstract: In this talk I will the concept of mean field games by using some simple examples. It turns out these examples can be viewed through the lens of classical nonlinear transport equations, such as the Burgers equation. I will discuss how the theory of entropy solutions might (or might not!) relate to the problem of selecting among multiple Nash equilibria for a mean field game.

9:35 am – 10:00 am Roshini Gallage, University of Oklahoma

Nonlinear Stochastic Differential Equations with Continuously Distributed Delay

Abstract: Stochastic delay differential equations (SDDEs) are systems of differential equations with a time lag in a noisy or random environment. The stochastic delay differential equation concept is widely used among modern day research due to rapid progress and understanding of stochastic delay differential equations and systems. Much research has been done using discrete delay where the dynamics of a process at time t depend on the state of the process in the past after a single fixed time lag t. In this talk, processes with continuously distributed delay which depend on weighted averages of past states over the entire time lag interval [t-t,t] are discussed. We give generalized Khasminskii-type conditions which along with local Lipschitz conditions are sufficient to guarantee the existence of a unique solution of certain n-dimensional nonlinear SDDEs with continuously distributed delay. Further, we give conditions under which Euler-Maruyama numerical approximations of such nonlinear SDDEs converge in probability to their exact solutions. Joint work with Dr. Harry Randolph Hughes.

10:00 am – 10:30 am **Coffee break** **Room 340**

Session B: Room 344

10:30 am – 10:55 am Qin Sheng, Baylor University

Splitting methods in the approximation of differential equation solutions

Abstract: While splitting methods have been widely used for approximating solutions of linear and nonlinear differential equations nowadays, their basic ideas can be traced back to Henry F. Baker (1866-1956), John E. Campbell (1862-1924) and Felix Hausdorff (1868-1942). The backbone of any modern splitting method is to decompose a differential equation problem to certain subproblems, then to combine corresponding subsolutions obtained for a reasonable approximation of the solution of the original differential equation problem. This short presentation recalls the phenomenal work done by aforementioned pioneers, and highlights several most popular splitting strategies in modern computations. Graduate and undergraduate students are very welcome to this talk.

11:00 am – 11:25 am Pedro Takemura, Baylor University

Geometric Herz Spaces and Poisson Kernels

Abstract: Classical Herz spaces (in the Euclidean setting) are defined in the spirit of the Littlewood-Paley theory, in which one localizes functions on dyadic annuli (centered at the origin), measures their size using Lebesgue spaces, and assembles these pieces all together using a weighted sequence quasinorm. In this talk we introduce a new class of Herz-type spaces, referred to as Geometric Herz spaces. This new brand generalizes its classical counterpart in that now, the functions under consideration are localized on tubular regions determined by a fixed d-Ahlfors regular set. Remarkably, these spaces are stable under the real method of interpolation. This permits us to obtain crucial boundedness and embedding results, which open the door for effectively employing Poisson kernels to treat the Dirichlet problem with data taken on Geometric Herz spaces. This is joint work with Marius Mitrea, Marcus Laurel, and Jesus Cruz-Lugo.

11:30 am – 11:55 pm Ellie Matter, Baylor University

Cluster Formation in Iterated Mean Field Games

Abstract: We look at a simple first-order Mean Field Game that gives players incentive to congregate. With a short enough time horizon, this type of game has a unique Nash Equilibrium given an initial distribution of players. Since the game is played only for a short time, we consider iterating the game, each new iteration starting at the final distribution of the previous game. We prove that after sufficiently many iterations, the players do congregate in tighter and tighter clusters and show where these clusters form.

12:00 pm- 2:00 pm **Lunch at Penland**

Session C: Room 344

2:00 pm – 2:25 pm Goong Chen, Texas A&M

Animal Shapes, Modal Analysis and Visualization

Abstract: Eigenfunctions and eigenvalues of physical systems and engineering structures can reveal many of the system's fundamental features and, therefore, become a basis for the study of inverse problems. In a recent series of papers published in J. Geometric Analysis, the speaker and his collaborators take a direct problem point of view; namely, given the shapes of animals, can we see the patterns of their motions or behaviors through their eigenmode analysis? This modal analysis has never been done for living animals, to the best of our knowledge. Our modal analysis emphasizes dynamics, which is achieved by visualization through video animation by incorporating the time-harmonic dependence of the eigenmodes. Furthermore, we intend our modal analysis to be more realistic by encompassing the situation of the presence of a floor. Certain physical interpretations of the motion patterns from modal analysis are made. In addition, by visualization, one can see that symmetry plays an important role in motion patterns. One of our main conclusions is that shapes alone can usefully reflect or explain some animal's behavior or motion patterns.

2:30 pm – 2:55 pm Huan Xu, The University of Texas at San Antonio

On a classification of steady solutions to two-dimensional Euler equations

Abstract: We present a classification of steady solutions to two-dimensional incompressible Euler equations in terms of the set of flow angles. The first main result asserts that the set of flow angles of any bounded steady flow in the whole plane must be the whole circle unless the flow is a parallel shear flow. In an infinitely long horizontal strip or the upper half-plane and under tangential boundary conditions, there exists an additional class of steady flows for which the set of flow angles is either the upper or lower closed semicircles. This type of flows is proved to be the class of non-shear flows

that have the least curved streamlines. We also present an application of our classification result to the structural stability of any shear flow whose all stagnation points are not inflection points, including Poiseuille flow as a special case.

3:00 pm – 3:25 pm Jesus Cruz-Lugo, Baylor University

A Fatou Theorem for Weakly Elliptic Systems in the Plane

Abstract: We establish a Fatou-type theorem in relation to the L^p-Dirichlet problem in the upper-half plane formulated for homogeneous, second-order, constant real coefficient, L^p- weakly elliptic 2×2 systems in the plane satisfying the condition of not being equivalent to any pathological system of the form $\overline{\partial^2} + \eta\partial^2$ with $\eta \in [0,1)$. Here, $\overline{\partial^2}$ and ∂^2 denote the squares of the Cauchy-Riemann operator and its conjugate, respectively. This is joint work with Marius Mitrea.

3:30 pm – 3: 55 pm Mohammad Mahabubur Rahman, Sam Houston State University

On a regularity criterion for the three-dimensional Hall-magnetohydrodynamics system

Abstract: The exploration of the global regularity criterion in terms of $\nabla b_h$for the three-dimensional Hall-magnetohydrodynamics system is extremely difficult due to the presence of the Hall term. This talk presents a proof in this direction.

Session D: Room 324

2:00 pm – 2:25 pm Henry Foust* and Youn-Sha Chan, University of Houston-Downtown

Toward a Simpler model for laminar flame speed associated with Solid Combustion

Abstract: In this presentation, we'll provide a primer on combustion and combustion related to solids. We'll go on to discuss two approaches to developing a model for the laminar flame speed associated with a solid combustion. These two approaches are discussed below.

Williams who was a pioneer in solid combustion determined a model for laminar flame speed of certain classes of solids by assuming the system can be modeled as a system of mist. This effort included the following sub-models and starts from the model originally developed by Mallard and Le Chatelier:

1. A model for the particle shrinkage
2. A model relating particle size to particle density

A more recent effort is the work of Goroshin and his students. In the following, Goroshin and Bidabadi determine models for a non-dimensional flame speed (kappa) for two cases:

1. Fuel lean systems
2. Fuel rich systems

Please note some of the goals of the planned future research include:

1. Determining the effects of particle shape and size on laminar flame speed
2. Determining the effects of binary mixtures of size on laminar flame speed
3. Appropriateness of using a mist model with modifications to account for particle shape and size as a model for laminar flame speed

2:30 pm – 2:55 pm Maria P. Fernando* and S. M. Mallikarjunaiah, Texas A&M University-Corpus Christi

On modeling brittle fracture within the context of strain-limiting theories of elasticity

Abstract: In this talk, first I will emphasize the need for a new class of constitutive relations to characterize the response of elastic solids. The class of models considered allows a nonlinear relationship between the primitive mechanical variables that have hitherto defined competent explanations about the evolution of cracks. The novel theory leads to a nonlinear system of partial differential equations that presents several challenges. Then, I will explain the analysis and implementation of a staggered iterative scheme for a two-field variational inequality system obtained from a regularized model for the quasi-static crack evolution problem. The proposed method involves a continuous Galerkin-type finite element discretization of two subproblems with each containing stabilization terms to improve the efficiency and robustness of the overall algorithm. Finally, I will present some interesting results from the direct numerical simulations of quasi-static crack evolution in elastic solids.

3:00 pm – 3:25 pm S. M. Mallikarjunaiah, Texas A&M University-Corpus Christi

On a New Theory for Brittle Fracture in Porous Elastic Solids whose Material Moduli Depend Upon the Density

Abstract: In this talk, I will discuss about mathematical and mechanical aspects of porous elastic solids using a novel class of nonlinear implicit constitutive theories. Such a description is necessary to characterize the response of many materials such as rocks, concrete, bones, and high-strength titanium alloys. Starting from the implicit theory, I will derive a sub-class of nonlinear relations wherein the stress and the linearized strain appear linearly, and one can't obtain such constitutive relations starting from the classical theory of linear elasticity. Then, a mixed variational formulation for the three field variables such as displacement, deviatoric stress, and spherical stress will be presented for a static and quasi-static crack problem. The existence theorem for the well-posedness of the regularized problem will be discussed. Finally, I will present some interesting finite element simulation results that can be directly correlated with the response of real-world engineering materials.

3:30 pm – 3:55 pm Dipesh Baral, Washington State University

A Stochastic Differential Equation for Active Transport

Abstract**:** We formulate a stochastic differential equation by deriving a Fokker-Planck equation from the continuous limit of the Chapman-Kolmogorov equation of a discrete bivariate stochastic process. We explore some statistical properties of the equation, like transient super diffusion, ephemeral non-Gaussian displacement distribution, non-monotonic evolution of Non-Gaussian Parameter (NGP), and tempered power-law distributed displacement steps. These properties have been observed in many crucial active transport processes, like the pursuit of foreign particles in the bloodstream by white blood cells, the transmission of impulses in a neuron, and the secretion of hormones by endocrine glands. Many chemically powered nanoparticles also exhibit these properties. As such, we present our system as a model for active transport. The equation can be solved numerically using the Euler- Maruyama method. Analytical results exist for special cases. Additionally, using the continuous time random walk formalism for a persistent random walk, we show that the Telegraph equation can approximate the system.

3:55 pm – 4:20 pm **Coffee break Room 340**

Session E: Room 344

4:20 pm – 4:45 pm Eduardo Servin Torres, Baylor University

A study of finite difference approximations of the 2D Kawarada equation with cross derivative terms

Abstract: Nonlinear Kawarada partial differential equations have been utilized in modeling sophisticated fuel combustion processes in the energy industry in decades. It has been challenging to introduce suitable finite difference approximations of the equations due to their strong quenching and degeneracy singularities, though without the presence of cross partial derivatives. Certain ambiguity and uncertainty have been exist in corresponding analysis. This talk targets at a new approximation strategy for the modeling problem without cancelling the cross partial derivative term which plays an important role in the description of the fuel rotations and movements inside, say, a combustion chamber. Numerical stability in the von Neumann sense will be proved without freezing the nonlinear reaction term involved. Simulation illustrations will be demonstrated. The study is in collaboration with Dr. Q. Sheng, Baylor University.

4:50 pm – 5: 15 pm Haseeb Ansari, University of Houston

Uniform Boundedness and Long-time behavior of Solutions to an SI model with Intermittent Treatment

Abstract: We study a dynamic SI (Susceptible-Infected) model with an intermittent treatment term. Results are given for the cases when no components diffuse, only 1 diffuses, and all components diffuse. In each case, we prove uniform boundedness of solutions and investigate their long-time behavior.

5:30 pm – 7:00 pm **Dinner**