1	2	3	4	5	6	7	8	9	10	11	12	Spring 2023

NAME______________________________ MTH 1322 - CALCULUS II

INSTRUCTOR______________________________

DIRECTIONS: You must show enough of your work so that the reader can follow what you did. No calculator of any sort is allowed on the exam. Simplify your answers. Your final answers may contain exact expressions such as $\ln 2$, e^{-1}, or $\pi/4$. However, expressions such as $\ln 1$, $e^{\ln 2}$, or $\sin(\pi/4)$ would have to be simplified (evaluated).

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1. Evaluate each of the following integrals.

   (a) $\displaystyle\int x \ln x \, dx$

   (b) $\displaystyle\int \frac{dx}{x \ln x}$

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2. Evaluate $\displaystyle\int \frac{dx}{(1+x^2)^{3/2}}$.

3. Evaluate $\displaystyle\int \frac{2x-1}{x^2-3x+2}\,dx$.

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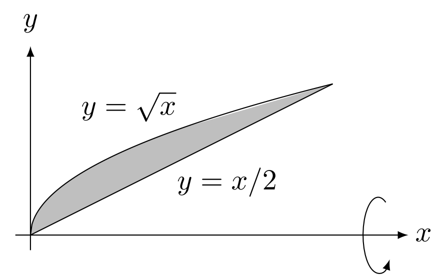
4. Evaluate each of the following improper integrals. For full credit, write limits.

(a) $\displaystyle\int_2^{\infty} \frac{dx}{x^4}$

(b) $\displaystyle\int_0^{\infty} xe^{-2x}\,dx$

5. Find the volume of the solid obtained by rotating the region shown about the x-axis.



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6. Solve the initial value problem $\dfrac{dy}{dx} = \dfrac{x}{y}$, $y(1) = -2$.

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7. (a) Determine the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{1}{4^n}$.

(b) Verify that $0.9999999\ldots = 1$ by expressing the left side as a geometric series and determining the sum of the series.

8. Determine for each series whether it converges or diverges. Justify your answers.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^5+1}$

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{2^n}$

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9. Show that the series $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{1}{\sqrt{n^2+1}}$ converges conditionally.

10. Find the Taylor polynomial $T_2(x)$ centered at $a = 4$ for the function $f(x) = \sqrt{x}$.

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11. Express the function $F(x) = \displaystyle\int_0^x \frac{\sin t}{t}\, dt$ as a power series in x.

Hint: You may start by using the known Maclaurin series for $\sin t$.

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12. Use Euler's formula to express each of the following complex numbers in $a + bi$ form.

(a) $4e^{\frac{2\pi}{3}i}$

(b) $ie^{-\frac{\pi}{4}i}$

Table of Trigonometric Integrals

$$\int \tan\theta\, d\theta = \ln|\sec\theta| + C = -\ln|\cos\theta| + C$$

$$\int \cot\theta\, d\theta = -\ln|\csc\theta| + C = \ln|\sin\theta| + C$$

$$\int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + C$$

$$\int \csc\theta\, d\theta = -\ln|\csc\theta + \cot\theta| + C$$

$$\int \sin^n\theta\, d\theta = -\frac{1}{n}\sin^{n-1}\theta\cos\theta + \frac{n-1}{n}\int \sin^{n-2}\theta\, d\theta$$

$$\int \cos^n\theta\, d\theta = \frac{1}{n}\cos^{n-1}\theta\sin\theta + \frac{n-1}{n}\int \cos^{n-2}\theta\, d\theta$$

$$\int \tan^n\theta\, d\theta = \frac{1}{n-1}\tan^{n-1}\theta - \int \tan^{n-2}\theta\, d\theta$$

$$\int \cot^n\theta\, d\theta = -\frac{1}{n-1}\cot^{n-1}\theta - \int \cot^{n-2}\theta\, d\theta$$

$$\int \sec^n\theta\, d\theta = \frac{1}{n-1}\sec^{n-2}\theta\tan\theta + \frac{n-2}{n-1}\int \sec^{n-2}\theta\, d\theta$$

$$\int \csc^n\theta\, d\theta = -\frac{1}{n-1}\csc^{n-2}\theta\cot\theta + \frac{n-2}{n-1}\int \csc^{n-2}\theta\, d\theta$$