1. Evaluate each of the following integrals.

(a) \( \int x \ln x \, dx \)

(b) \( \int \frac{dx}{x \ln x} \)

2. Evaluate \( \int \frac{dx}{(1 + x^2)^{3/2}} \).
3. Evaluate \( \int \frac{2x - 1}{x^2 - 3x + 2} \, dx \).

4. Evaluate each of the following improper integrals. For full credit, write limits.

   (a) \( \int_{2}^{\infty} \frac{dx}{x^4} \)

   (b) \( \int_{0}^{\infty} xe^{-2x} \, dx \)
5. Find the volume of the solid obtained by rotating the region shown about the $x$-axis.

6. Solve the initial value problem \( \frac{dy}{dx} = \frac{x}{y}, \quad y(1) = -2 \).

7. (a) Determine the sum of the series \( \sum_{n=0}^{\infty} \frac{1}{4^n} \).

(b) Verify that \( 0.9999999\ldots = 1 \) by expressing the left side as a geometric series and determining the sum of the series.
8. Determine for each series whether it converges or diverges. Justify your answers.

(b) $\sum_{n=1}^{\infty} \frac{n}{n^5 + 1}$

9. Show that the series $\sum_{n=0}^{\infty} (-1)^n \frac{1}{\sqrt{n^2 + 1}}$ converges conditionally.
10. Find the Taylor polynomial $T_2(x)$ centered at $a = 4$ for the function $f(x) = \sqrt{x}$.

11. Express the function $F(x) = \int_0^x \frac{\sin t}{t} \, dt$ as a power series in $x$.
   
   Hint: You may start by using the known Maclaurin series for $\sin t$.

12. Use Euler’s formula to express each of the following complex numbers in $a + bi$ form.
   
   (a) $4e^{\frac{2\pi}{3}i}$
   
   (b) $ie^{-\frac{\pi}{4}i}$
Table of Trigonometric Integrals

\[ \int \tan \theta \, d\theta = \ln | \sec \theta | + C = -\ln | \cos \theta | + C \]

\[ \int \cot \theta \, d\theta = -\ln | \csc \theta | + C = \ln | \sin \theta | + C \]

\[ \int \sec \theta \, d\theta = \ln | \sec \theta + \tan \theta | + C \]

\[ \int \csc \theta \, d\theta = -\ln | \csc \theta + \cot \theta | + C \]

\[ \int \sin^n \theta \, d\theta = -\frac{1}{n} \sin^{n-1} \theta \cos \theta + \frac{n-1}{n} \int \sin^{n-2} \theta \, d\theta \]

\[ \int \cos^n \theta \, d\theta = \frac{1}{n} \cos^{n-1} \theta \sin \theta + \frac{n-1}{n} \int \cos^{n-2} \theta \, d\theta \]

\[ \int \tan^n \theta \, d\theta = \frac{1}{n-1} \tan^{n-1} \theta - \int \tan^{n-2} \theta \, d\theta \]

\[ \int \cot^n \theta \, d\theta = -\frac{1}{n-1} \cot^{n-1} \theta - \int \cot^{n-2} \theta \, d\theta \]

\[ \int \sec^n \theta \, d\theta = \frac{1}{n-1} \sec^{n-2} \theta \tan \theta + \frac{n-2}{n-1} \int \sec^{n-2} \theta \, d\theta \]

\[ \int \csc^n \theta \, d\theta = -\frac{1}{n-1} \csc^{n-2} \theta \cot \theta + \frac{n-2}{n-1} \int \csc^{n-2} \theta \, d\theta \]