1	2	3	4	5	6	7	8	9	10	11	12	Spring 2022

NAME______________________________________ MTH 1322 - CALCULUS II

INSTRUCTOR________________________________

DIRECTIONS: You must show enough of your work so that the reader can follow what you did. No calculator of any sort is allowed on the exam. Simplify your answers. Your final answers may contain exact expressions such as $\ln 2$, e^{-1}, or $\pi/4$. However, expressions such as $\ln 1$, $e^{\ln 2}$, or $\sin(\pi/4)$ would have to be simplified (evaluated).

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1. Evaluate $\displaystyle\int_0^1 x^2 e^x \, dx$.

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2. Evaluate $\displaystyle\int \frac{dx}{(1-x^2)^{3/2}}$.

3. (a) Evaluate $\displaystyle\int \frac{dx}{x(x+1)}$.

(b) Use your answer from part (a) to evaluate the improper integral $\displaystyle\int_1^\infty \frac{dx}{x(x+1)}$.

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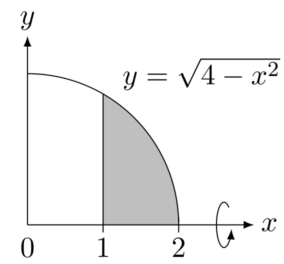
4. Use L'Hôpital's rule to evaluate each of the following limits.

(a) $\displaystyle\lim_{x\to 0} \frac{x^2}{1-\cos x}$

(b) $\displaystyle\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}}$

5. Find the volume of the solid obtained by rotating the shaded region about the x-axis.



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6. Solve the initial value problem $\dfrac{dy}{dx} = y + 1, \quad y(0) = 1.$

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7. Write the geometric series $\dfrac{1}{2} + \dfrac{3}{8} + \dfrac{9}{32} + \dfrac{27}{128} + \dfrac{81}{512} + \cdots$ in Σ-notation and find its sum.

8. Use the integral test to determine whether the series $\sum_{n=1}^{\infty} ne^{-n^2}$ converges.

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9. Use a comparison test to determine whether each of the following series converges.

(a) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{1+n^3}}$

(b) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{1+n^3}}$

10. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{1}{n} x^n$. Pay attention to the endpoints.

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11. Let $f(x) = \cos(x^3)$.

   (a) Write out the first four nonzero terms of the Maclaurin series for $f(x)$.
   Hint: Start with the known Maclaurin series for $\cos x$.

   (b) Use your answer from part (a) to find $f^{(6)}(0)$.

12. Use Euler's formula to write each of the following complex numbers in $a + bi$ form.

(a) $e^{-\frac{\pi}{2}i}$

(b) $-2ie^{\frac{2\pi}{3}i}$

Table of Trigonometric Integrals

$$\int \tan\theta\, d\theta = \ln|\sec\theta| + C = -\ln|\cos\theta| + C$$

$$\int \cot\theta\, d\theta = -\ln|\csc\theta| + C = \ln|\sin\theta| + C$$

$$\int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + C$$

$$\int \csc\theta\, d\theta = -\ln|\csc\theta + \cot\theta| + C$$

$$\int \sin^n\theta\, d\theta = -\frac{1}{n}\sin^{n-1}\theta\cos\theta + \frac{n-1}{n}\int \sin^{n-2}\theta\, d\theta$$

$$\int \cos^n\theta\, d\theta = \frac{1}{n}\cos^{n-1}\theta\sin\theta + \frac{n-1}{n}\int \cos^{n-2}\theta\, d\theta$$

$$\int \tan^n\theta\, d\theta = \frac{1}{n-1}\tan^{n-1}\theta - \int \tan^{n-2}\theta\, d\theta$$

$$\int \cot^n\theta\, d\theta = -\frac{1}{n-1}\cot^{n-1}\theta - \int \cot^{n-2}\theta\, d\theta$$

$$\int \sec^n\theta\, d\theta = \frac{1}{n-1}\sec^{n-2}\theta\tan\theta + \frac{n-2}{n-1}\int \sec^{n-2}\theta\, d\theta$$

$$\int \csc^n\theta\, d\theta = -\frac{1}{n-1}\csc^{n-2}\theta\cot\theta + \frac{n-2}{n-1}\int \csc^{n-2}\theta\, d\theta$$