1	2	3	4	5	6	7	8	9	10	11	12	Fall 2023

NAME____ MTH 1322 - CALCULUS II

INSTRUCTOR____

DIRECTIONS: For full credit you must show enough of your work so that the reader can follow what you did. No calculator of any sort is allowed on the exam. Simplify your answers. Your final answers may contain exact expressions such as $\ln 2$, e^{-1}, or $\pi/4$. However, expressions such as $\ln 1$, $e^{\ln 2}$, or $\sin(\pi/4)$ would have to be simplified (evaluated).

---

1. Evaluate $\int x^2 \sin 2x \, dx$.

---

2. Evaluate $\int \dfrac{2x-1}{x(x-1)} dx$.

3. Evaluate the definite integral $\int_0^{\pi/2} \cos^3 x \, dx$.

---

4. Evaluate the improper integral, $\int_0^{\infty} \frac{2x}{(1+x^2)^2} \, dx$.

5. Use the comparison test to determine whether the integral converges, $\int_0^1 \frac{1}{\sqrt{x}+x^2}\,dx$.

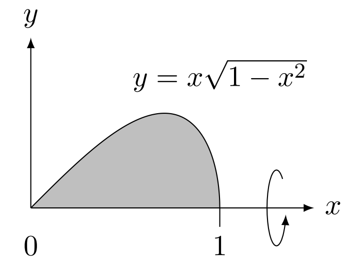
---

6. Use L'Hôpital's rule to evaluate the limit.

(a) $\lim_{x\to 0} \frac{e^x - 1}{\sin x}$

(b) $\lim_{x\to \infty} \frac{x^2}{e^x}$

7. Find the volume of revolution about the x-axis for the region under the curve $y = x\sqrt{1-x^2}$ over the interval $[0,1]$.



---

8. Solve the initial value problem $y^2 \dfrac{dy}{dx} = x^{-2}$, $y(1) = 0$.

9. Let $F(x) = \int_0^x \frac{1}{1+t^3}\,dt$. Use the Maclaurin series for $\frac{1}{1-x}$ to show that $F(x) = x - \frac{x^4}{4} + \frac{x^7}{7} - \ldots$.

---

10. Find the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(x-2)^n}{(2n)!}$.

11. Find the sum of the series or state that the series diverges and justify your answer.

(a) $\frac{8}{5} + \frac{8}{5^2} + \frac{8}{5^3} + \frac{8}{5^4} + \ldots$

(b) $\sum_{n=1}^{\infty} \frac{n}{3n-2}$

---

12. Determine whether the series $\sum_{n=1}^{\infty} \frac{\cos n}{n^3}$ converges absolutely, conditionally, or not at all.

Table of Trigonometric Integrals

$$\int \tan\theta\,d\theta = \ln|\sec\theta| + C = -\ln|\cos\theta| + C$$

$$\int \cot\theta\,d\theta = -\ln|\csc\theta| + C = \ln|\sin\theta| + C$$

$$\int \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta| + C$$

$$\int \csc\theta\,d\theta = -\ln|\csc\theta + \cot\theta| + C$$

$$\int \sin^n\theta\,d\theta = -\frac{1}{n}\sin^{n-1}\theta\cos\theta + \frac{n-1}{n}\int \sin^{n-2}\theta\,d\theta$$

$$\int \cos^n\theta\,d\theta = \frac{1}{n}\cos^{n-1}\theta\sin\theta + \frac{n-1}{n}\int \cos^{n-2}\theta\,d\theta$$

$$\int \tan^n\theta\,d\theta = \frac{1}{n-1}\tan^{n-1}\theta - \int \tan^{n-2}\theta\,d\theta$$

$$\int \cot^n\theta\,d\theta = -\frac{1}{n-1}\cot^{n-1}\theta - \int \cot^{n-2}\theta\,d\theta$$

$$\int \sec^n\theta\,d\theta = \frac{1}{n-1}\sec^{n-2}\theta\tan\theta + \frac{n-2}{n-1}\int \sec^{n-2}\theta\,d\theta$$

$$\int \csc^n\theta\,d\theta = -\frac{1}{n-1}\csc^{n-2}\theta\cot\theta + \frac{n-2}{n-1}\int \csc^{n-2}\theta\,d\theta$$