1	2	3	4	5	6	7	8	9	10	11	Fall 2022

NAME____ MTH 1322 - CALCULUS II

INSTRUCTOR____________________________________

DIRECTIONS: You must show enough of your work so that the reader can follow what you did. No calculator of any sort is allowed on the exam. Simplify your answers. Your final answers may contain exact expressions such as $\ln 2$, e^{-1}, or $\pi/4$. However, expressions such as $\ln 1$, $e^{\ln 2}$, or $\sin(\pi/4)$ would have to be simplified (evaluated).

---

1. (a) Evaluate $\int xe^{-2x}\,dx$.

   (b) Evaluate the improper integral $\int_0^\infty xe^{-2x}\,dx$.

---

2. Evaluate $\int \frac{dx}{\sqrt{4+x^2}}$.

3. Evaluate $\displaystyle\int \frac{1+x^2}{x^2(1+x)}\, dx$.

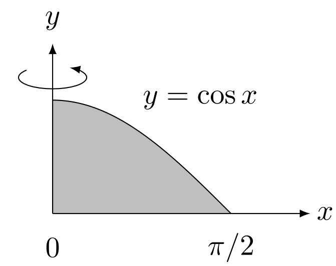
---

4. Use a comparison test to determine whether each of the following improper integrals converges.

   (a) $\displaystyle\int_1^{\infty} \frac{dx}{\sqrt{1+x^3}}$

   (b) $\displaystyle\int_1^{\infty} e^{-x^3}\, dx$ *Hint*: $x^3 \geq x$ for $x \geq 1$

5. Use the shell method to find the volume of the solid obtained by rotating the shaded region about the y-axis.



---

6. Solve the initial value problem $\dfrac{dy}{dt} = -2\sqrt{y}, \quad y(0) = 4.$

---

7. Write the geometric series $\dfrac{1}{2} - \dfrac{1}{6} + \dfrac{1}{18} - \dfrac{1}{54} + \dfrac{1}{162} - \dfrac{1}{486} + \cdots$ in Σ-notation and find its sum.

8. (a) Use the integral test to show that the series $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ diverges.

(b) Does the series $\sum_{n=2}^{\infty} (-1)^n \frac{1}{n \ln n}$ converge conditionally? Justify your answer.

---

9. Determine whether each of the following series converges or diverges. Clearly state which tests you are using.

(a) $\sum_{n=1}^{\infty} \frac{n^2}{\sqrt{1+n^5}}$

(b) $\sum_{n=1}^{\infty} \frac{3^n}{(2n)!}$

10. (a) Expand the function $g(x) = \dfrac{1}{1+x^2}$ as a power series (Maclaurin series).

Hint: You may take the power series expansion of $\dfrac{1}{1-x}$ for granted.

(b) For which x is the series expansion in part (a) valid? No justification is needed.

(c) Expand the function $f(x) = \tan^{-1} x$ as a power series (Maclaurin series) by integrating the series from part (a) term by term.

---

11. Use Euler's formula to write each of the following complex numbers in the form $a + bi$.

(a) $e^{2\pi i/3}$

(b) $\left(\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}\, i\right)^{12}$

Table of Trigonometric Integrals

$$\int \tan\theta\, d\theta = \ln|\sec\theta| + C = -\ln|\cos\theta| + C$$

$$\int \cot\theta\, d\theta = -\ln|\csc\theta| + C = \ln|\sin\theta| + C$$

$$\int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + C$$

$$\int \csc\theta\, d\theta = -\ln|\csc\theta + \cot\theta| + C$$

$$\int \sin^n\theta\, d\theta = -\frac{1}{n}\sin^{n-1}\theta\cos\theta + \frac{n-1}{n}\int \sin^{n-2}\theta\, d\theta$$

$$\int \cos^n\theta\, d\theta = \frac{1}{n}\cos^{n-1}\theta\sin\theta + \frac{n-1}{n}\int \cos^{n-2}\theta\, d\theta$$

$$\int \tan^n\theta\, d\theta = \frac{1}{n-1}\tan^{n-1}\theta - \int \tan^{n-2}\theta\, d\theta$$

$$\int \cot^n\theta\, d\theta = -\frac{1}{n-1}\cot^{n-1}\theta - \int \cot^{n-2}\theta\, d\theta$$

$$\int \sec^n\theta\, d\theta = \frac{1}{n-1}\sec^{n-2}\theta\tan\theta + \frac{n-2}{n-1}\int \sec^{n-2}\theta\, d\theta$$

$$\int \csc^n\theta\, d\theta = -\frac{1}{n-1}\csc^{n-2}\theta\cot\theta + \frac{n-2}{n-1}\int \csc^{n-2}\theta\, d\theta$$