1	2	3	4	5	6	7	8	9	10	11	12	13	Fall 2023

NAME______________________________________ MTH 1321 - CALCULUS I

INSTRUCTOR__________________________________

DIRECTIONS: You must show enough of your work so that the grader can follow what you did. No calculator of any sort is allowed on the exam. Your final answers may contain exact expressions such as $\sqrt{2}$, $\ln 2$, e^{-1}, or $\pi/4$. However, expressions such as $\sqrt{4}$, $\ln 1$, $e^{\ln 2}$, or $\sin(\pi/4)$ would have to be simplified (evaluated).

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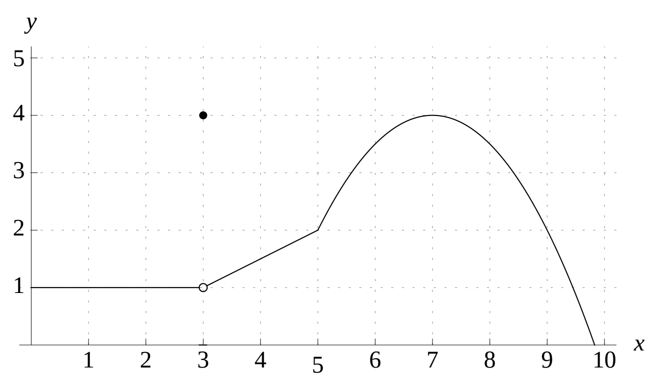
1. Use the graph of the function f to find each of the following values or write DNE (does not exist):



(a) $f(3)$

(b) $\lim_{x\to 3} f(x)$

(c) $\lim_{x\to 5} \dfrac{f(x)}{x}$

(d) $f'(4)$

(e) $\lim_{h\to 0} \dfrac{f(5+h)-f(5)}{h}$

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2. Let $f(x) = \dfrac{1}{x}$, $x \neq 0$. Use the limit definition of the derivative to show that $f'(x) = -\dfrac{1}{x^2}$.

3. Evaluate the following limits:

   (a) $\lim_{t \to 0} \dfrac{1}{1 + \dfrac{\sin t}{t}}$

   (b) $\lim_{x \to \infty} \dfrac{\sqrt{4x^2 + 7}}{9x + 4}$

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4. Use the derivative rules we studied to compute the following derivatives:

   (a) $\dfrac{d}{dz} \left(\dfrac{1 - z}{1 + z} \right)^7$

   (b) $\dfrac{d}{dx} \ln(1 + e^{2x})$

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5. Find an equation of the line that is tangent to the curve $xe^y + x^2y^2 = 1$ at the point $(1, 0)$.

   Hint: Use implicit differentiation to find the slope of the tangent line.

6. Gas is escaping from a spherical balloon at a rate of 2 cubic feet per minute. How fast is the radius decreasing when the radius is 5 feet? Give your answer with the correct units.

   Hint: The volume of a sphere of radius r is $V = \dfrac{4\pi r^3}{3}$.

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7. Find the maximum and minimum values of the function $f(x) = \sin(x) + \cos(x)$ on the interval $[0, \pi]$.

8. Determine the open intervals on which the function $f(x) = x^4 - 4x^3 + 10$ is concave up or down and find the points of inflection.

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9. Find the point on the graph $y = \sqrt{x}$ that is closest to the point $(2, 0)$.

   Hint: Minimize the square of the distance from a point on the graph $y = \sqrt{x}$ to the point $(2, 0)$. Don't forget to justify that the minimum is indeed a minimum.

10. Evaluate the following definite integrals:

(a) $\displaystyle\int_1^2 \left(x + x^{-1}\right)\, dx$

(b) $\displaystyle\int_0^{\pi/4} \sec^2(\theta)\, d\theta$

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11. Evaluate the following indefinite integrals:

(a) $\displaystyle\int e^{-2x}\, dx$

(b) $\displaystyle\int \frac{x}{\sqrt{1+x^2}}\, dx$

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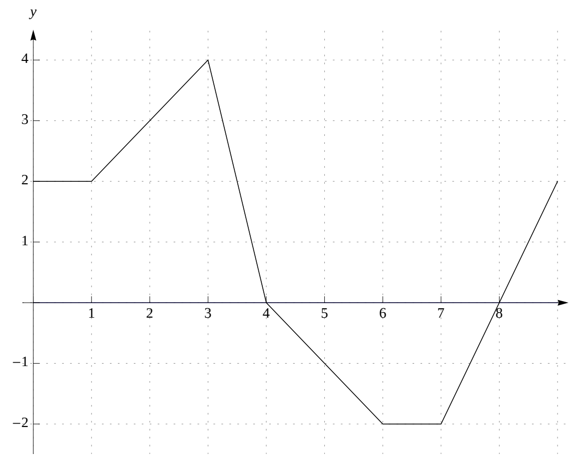
12. The velocity of an accelerating motorcycle at 1-second intervals is given in the following table:

t (s)	0	1	2	3	4	5	6
v (ft/s)	0	6	14	23	30	36	45

(a) Estimate the acceleration of the motorcycle at $t = 3$ seconds. Give your answer with the correct units.

(b) Estimate the distance traveled by the motorcycle during the first 6 seconds using the midpoint approximation M_3. Give your answer with the correct units.

13. Let $f(x) = \int_0^x g(t)\,dt$, where g is the function given by the graph below.



(a) Find $f'(4)$ and name the theorem you used to justify your answer.

(b) Use geometry to find $f(4)$.

(c) Find the maximum value of $f(x)$ on the interval $[0, 6]$. Justify your answer.