1	2	3	4	5	6	7	8	9	10	11	12	Spring 2021

NAME____ MTH 1322 - CALCULUS II

INSTRUCTOR____

DIRECTIONS: You must show enough of your work so that the reader can follow what you did. No calculator of any sort is allowed on the exam. Simplify your answers. Your final answers may contain exact expressions such as $\ln 2$, e^{-1}, or $\pi/4$. However, expressions such as $\ln 1$, $e^{\ln 2}$, or $\sin(\pi/4)$ would have to be simplified (evaluated).

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1. (a) Use integration by parts to evaluate $\int \ln x \, dx$.

   (b) Use your answer from part (a) to evaluate the improper integral $\int_0^2 \ln x \, dx$.
   Hint: You may have to use L'Hospital's rule.

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2. Use a trigonometric substitution to evaluate $\int \sqrt{1+x^2} \, dx$.
   Hint: You may consult the table of trigonometric integrals on the last page of the exam.

3. (a) Use a partial fraction decomposition to evaluate $\displaystyle\int \frac{dx}{x^2+4x+3}$.

(b) Does the improper integral $\displaystyle\int_1^{\infty} \frac{dx}{x^2+4x+3}$ converge? Justify your answer.

Hint: You do not need part (a) to answer the question in part (b).

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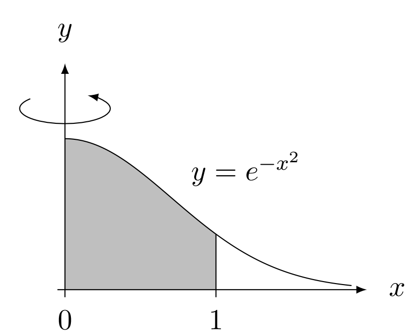
4. Use the shell method to find the volume of the solid obtained by rotating the shaded region about the y-axis.



5. (a) Express the arc length of the parabola $y = \frac{1}{2}x^2$ over $[0, 1]$ as an integral (but do not evaluate).

(b) Consider the surface obtained by rotating the parabola $y = \frac{1}{2}x^2$ over $[0, 1]$ about the x-axis. Express the surface area as an integral (but do not evaluate).

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6. Solve the initial value problem $y' = -xy^2, \quad y(0) = 2$.

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7. (a) Find the sum of the geometric series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4^n}$.

(b) Find the sum of the alternating series $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)!}$ to within an error of at most 0.01 $(= 1/100)$.

8. Use a comparison test to determine whether the series converges or diverges.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{3^n + 1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^4 + 1}}$

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9. Determine whether the series converges or diverges. Clearly state which tests you are using.

(a) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{n}{n+1}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{n!}$

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10. Compute the Maclaurin polynomial $T_2(x)$ for the function $f(x) = \sqrt{1 + x^2}$.

11. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n2^n}$. Pay attention to the endpoints.

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12. Find the Maclaurin series of the function $f(x) = \int_0^x \sin(t^2)\, dt$.

Hint: First write the Maclaurin series of $\sin(t^2)$ and then integrate term-by-term.

Table of Trigonometric Integrals

$$\int \tan x\,dx = \ln|\sec x| + C = -\ln|\cos x| + C$$

$$\int \cot x\,dx = -\ln|\csc x| + C = \ln|\sin x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$

$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x\sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx$$

$$\int \tan^n x\,dx = \frac{1}{n-1}\tan^{n-1} x - \int \tan^{n-2} x\,dx$$

$$\int \cot^n x\,dx = -\frac{1}{n-1}\cot^{n-1} x - \int \cot^{n-2} x\,dx$$

$$\int \sec^n x\,dx = \frac{1}{n-1}\sec^{n-2} x\tan x + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$$

$$\int \csc^n x\,dx = -\frac{1}{n-1}\csc^{n-2} x\cot x + \frac{n-2}{n-1}\int \csc^{n-2} x\,dx$$