Please do not write in these boxes.

1	2	3	4	5	6	7	8	9	10	11	12	13	TOTAL

MTH 1321 Name:______________________________

Fall 2022 Departmental Final Exam Instructor:______________________________

NO CALCULATOR ALLOWED

Complete the following in the space provided. Show the steps or reasoning leading to your answer for full credit. Numerical expressions do not need to be simplified, but trig functions should be evaluated whenever they appear.

1. Consider the following table of values involving a continuous function f at various values of x.

x	$f(x)$	$\frac{f(x)-f(1)}{x-1}$
0.9	2.948683	0.513167
0.99	2.994987	0.501256
0.999	2.9995	0.500125
0.9999	2.99995	0.500013
1.0001	3.0005	5.0001
1.001	3.005	5.001
1.01	3.05	5.01
1.1	3.5	5.1

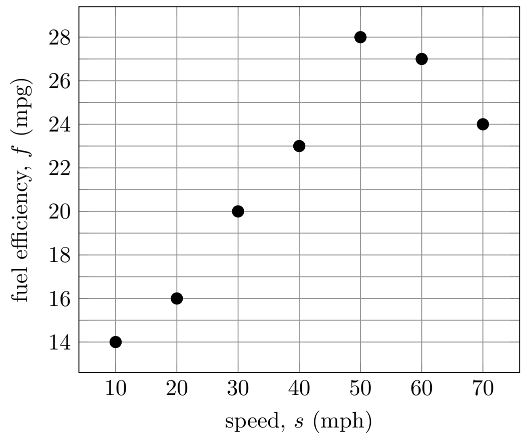
Use the information in the table to give a reasonable approximation for each of the following, or state that the quantity does not exist. In either case, briefly explain your reasoning.

(a) $\lim_{x\to 1} f(x)$

(b) $f(1)$

(c) $f'(1)$

2. Data for the fuel efficiency f (in miles per gallon, mpg) of an automobile at various speeds s (in miles per hour, mph) were collected from the driver's control panel and are summarized below.



Suppose that fuel efficiency is a differentiable function of speed for all speeds.

(a) Compute the average rate of change of the fuel efficiency for $10 \leq s \leq 70$. Include units.

(b) For speeds between 10 mph and 70 mph, what does the Mean Value Theorem say must be true about the fuel efficiency f? (Your answer should be specific to the context of this problem.)

(c) Estimate $\displaystyle\int_{10}^{70} f(s)\,ds$ via the midpoint method using the three subintervals $[10, 30]$, $[30, 50]$, and $[50, 70]$.

3. Evaluate the following limits.

   (a) $\lim\limits_{t \to 0} \dfrac{\tan(4t)}{8t}$

   (b) $\lim\limits_{x \to -\infty} \dfrac{1}{e^x + 1}$

4. A particle is in motion along the x-axis so that at time t, the velocity of the particle is given by $v(t) = \sqrt{t} + 1$. Furthermore, when $t = 1$, the particle is positioned at $x = 3$.

   (a) What is the acceleration of the particle when $t = 4$?

   (b) What is the position of the particle when $t = 0$?

5. Compute the following. You do not need to simplify your answer.

   (a) Let $y = \dfrac{e^x}{2-5x}$. Find $\dfrac{dy}{dx}$.

   (b) Let $f(x) = \cos^2(3^x)$. Find $f'(x)$.

6. Compute the following. You do not need to simplify your answer.

   (a) Let $f(x) = \sqrt[3]{x} \cdot \sec(2x)$. Find $f'(x)$.

   (b) Let $y = x^{\sin x}$. Find $\dfrac{dy}{dx}$. Write your answer in explicit form, i.e., solve for $\dfrac{dy}{dx}$ in terms of x.

7. Consider the implicit relation $x + y^2 = xy + 1$.

(a) Find $\dfrac{dy}{dx}$ at the point $(1, 0)$.

(b) Find $\dfrac{d^2y}{dx^2}$ at the point $(1, 0)$.

8. Compute the following:

(a) $\int (x^3+1)^2\,dx$

(b) $\int \frac{1}{1+9x^2}\,dx$

9. Compute the following:

(a) $\int_1^3 \frac{1}{5-x}\,dx$

(b) $\int_{\pi/4}^{\pi/2} \sin^3\theta\cos\theta\,d\theta$

10. You plug in your cell phone to charge its battery. At time t (in minutes), the function $c(t)$ tracks how fully charged the battery is (as a percentage) t minutes after you begin charging, i.e. when $c(t) = 100$, the battery is fully charged and when $c(t) = 0$ the battery is completely discharged.

(a) Suppose that exactly 10 minutes after you begin charging, the battery is at 15% charge and the charge in the battery is increasing by 1 percentage point per minute. Which of these best represents this information?

(A) $c(10) = 1$ and $c'(10) = 15$

(B) $c(10) = 15$ and $c'(10) = 1$

(C) $c'(10) = 15$ and $c''(10) = 1$

(D) $c(10) = 15$ and $\displaystyle\int_0^{10} c(t)\,dt = 1$

(E) $c'(10) = 15$ and $\displaystyle\int_0^{10} c(t)\,dt = 1$

(b) Suppose that, from the 10th minute of charging to the 20th minute of charging, the battery gained 12 percentage points of charge. Which of these best represents this information?

(A) $\dfrac{c(20) - c(10)}{20 - 10} = 12$

(B) $c'(10) = 12$

(C) $c'(10) = 12$ and $c'(20) = 12$

(D) $\displaystyle\int_{10}^{20} c'(t)\,dt = 12$

(E) $\displaystyle\int_{10}^{20} c(t)\,dt = 12$

(c) Suppose that, exactly at the 20th minute of charging, the rate at which the battery is charging is increasing. Which of these best represents this information?

(A) $c'(20) > 0$

(B) $c'(20) < 0$

(C) $c''(20) > 0$

(D) $c''(20) < 0$

(E) $\dfrac{c(20) - c(0)}{20 - 0} > 0$

(d) Suppose your battery is completely discharged when you plug it in and it takes 50 minutes for it to reach full charge. Over that time period, on average the battery gained 2 percentage points of charge per minute. Which of these best represents this information?

(A) $\dfrac{c(50) - c(0)}{50 - 0} = 2$

(B) $c'(50) = 2$

(C) $c'(0) = 2$ and $c'(50) = 2$

(D) $c''(50) = 2$

(E) $\displaystyle\int_0^{50} c'(t)\,dt = 2$

(e) Suppose that, at exactly the 15th minute of charging, the battery is charging, but doing so more slowly than it was a moment ago. Which of these best represents this information?

(A) $c(15) > 0$ and $c'(15) > 0$

(B) $c(15) > 0$ and $c'(15) < 0$

(C) $c(15) < 0$ and $c'(15) > 0$

(D) $c'(15) > 0$ and $c''(15) < 0$

(E) $c'(15) < 0$ and $c''(15) > 0$

11. Let $f(x) = \dfrac{\ln x}{x}$, $x > 0$. Find the x-coordinate of all local extrema of f and classify whether a local max or local min occurs there. Justify your answer.

12. Suppose that a function f is continuous for $0 \leq x \leq 5$ and differentiable for $0 < x < 5$. Suppose further that $f(0) = -2$ and $f(5) = 3$. Mark each of the following statements as true (T) or false (F). You do *not* need to justify your answer.

(a) $f(c) = 1$ for some c such that $0 < c < 5$.

(b) $f'(c) = 1$ for some c such that $0 < c < 5$.

(c) $f''(c) = 1$ for some c such that $0 < c < 5$.

(d) $f'(c) = 0$ for some c such that $0 < c < 5$.

(e) $\displaystyle\int_0^5 f(x)\,dx$ exists.

13. Let $A(x) = \int_0^x \sin t \, dt$ and $B(x) = \int_0^{\cos x} \sin t \, dt$.

(a) Find $A(\pi)$ and $A'(\pi)$.

(b) Find $B'(x)$.