Please do not write in these boxes.

1	2	3	4	5	6	7	8	9	10	11	12	13	TOTAL

MTH 1321 Name:_____

Fall 2017 Departmental Final Exam Instructor:_____

NO CALCULATOR ALLOWED

Complete the following in the space provided. Show the steps leading to your answer for full credit.

1. Compute the following limits.

   (a) $\lim_{x\to -3} \dfrac{x^2-9}{x^2-2x-15}$

   (b) $\lim_{x\to \infty} \dfrac{2x-\sqrt{x}}{x^3+2017}$

2. Suppose $f'(a)=0$. Mark each of the following statements as true (T) (meaning "must be true") or false (F) (meaning "could be false"). You do *not* need to justify your answer.

   (a) f is continuous at $x=a$.

   (b) The line tangent to $y=f(x)$ at $x=a$ is horizontal.

   (c) f has a local max or local min at $x=a$.

   (d) $\lim_{x\to a} \dfrac{f(x)-f(a)}{x-a} = 0$

   (e) f has an inflection point at $x=a$.

3. Suppose $f(x) = e^{\cos x}$, $g(x) = \ln(4 - 3x)$, and $h(x) = \tan^{-1}(x)$.

   (a) Find $[f(x)g(x)]'$. You do not need to simplify your answer.

   (b) Find $\left[h(\sqrt{x}\,)\right]'$. You do not need to simplify your answer.

4. Compute the following:

   (a) $\displaystyle\int \left(3\sec^2 x + \frac{1}{x^2} + 2^x - 4\right) dx$

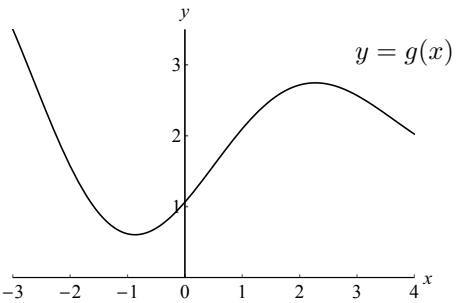
   (b) $\displaystyle\int_0^{\pi/2} \sin x \, dx$

5. Let $f(x) = \dfrac{x}{1+x^2}$.

(a) Find the absolute maximum and minimum values of f on the interval $[0, 3]$.

(b) Find the exact value of the area between the curve $y = f(x)$ and the x-axis for $0 \leq x \leq 3$.

6. (a) The graph of a twice-differentiable function g is shown below. Put these quantities in order from least to greatest: $g(0)$, $g'(-1)$, and $g''(2)$.



(b) Between classes you are standing in line to order a cup of coffee. When you have been in line for t minutes, there are a total of $p(t)$ people in the line.

Write a brief sentence interpreting $p(6) = 12$ and $p'(6) = 2$ in the context of this problem. Include units.

7. Let $f(x) = \dfrac{1}{x}$. Use the limit definition of the derivative to compute $f'(x)$.

8. Let $f(x) = x^4 + 4x^3$.

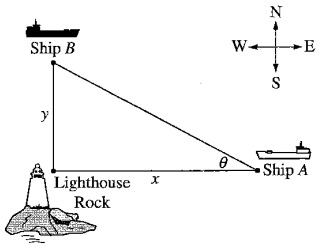
   (a) Find the intervals on which f is increasing/decreasing.

   (b) Find and classify all local extrema of f.

   (c) Find the intervals on which f is concave up/down.

   (d) Find all inflection points of f.

9. Ship A is traveling due west toward Lighthouse Rock at a speed of 15 km/hr. Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t as shown below.



Find the rate of change (in km/hr) of the distance between the two ships when $x = 4$ km and $y = 3$ km.

10. A marble rolls along the x-axis so that at any time $t > 0$, its velocity is given by $v(t) = 4 - 6t^2$. If the marble is at position $x = 7$ at time $t = 1$, what is the position of the marble at time $t = 2$?

11. Consider $f(x) = x + \dfrac{1}{x}$ on $1 \leq x \leq 2$. Find a value of c guaranteed by the Mean Value Theorem.

12. You ride your bike along a straight trail, recording your velocity $v(t)$ (in miles per hour) for selected values of t over the interval $0 \leq t \leq 1.5$ hours, as shown in the table below. For $0 < t \leq 1.5$, $v(t) > 0$.

t (hrs)	0	0.5	1.0	1.5
$v(t)$ (mi/hr)	0	12	8	18

(a) Use the data in the table to approximate your acceleration at time $t = 1.25$ hours. Include units.

(b) Approximate $\displaystyle\int_0^{1.5} v(t)\,dt$ using a right Riemann sum with three subintervals of equal length (i.e., the R_3 approximation) and values from the table. Include units.

(c) Interpret your answer in (b) in the context of this problem. Include units.

13. On Christmas Eve, snow begins to fall at a rate of $r(t) = t\sqrt{t^2+1}$ inches per hour, where t is measured in hours since midnight.

(a) Set up a function $A(x)$ which describes the accumulated amount (in inches) of snowfall x hours since midnight.

(b) How many inches of snow accumulate from midnight to 1 AM?

(c) Find the rate of change of the accumulation of snow at 1 AM. Include units.

(d) Find $A''(x)$ when $x = 1$. Interpret this value in the context of this problem. Include units.