

The Baylor Analysis Fest
*“From Operator Theory to Orthogonal
Polynomials, Combinatorics, and Number
Theory”*

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Abstracts

Sextic anharmonic oscillators and biconfluent Heun equations via differential Galois theory

Primitivo Acosta-Humánez

Instituto de Matematica, Universidad Autonoma de Santo Domingo, Dominican Republic

With certain constraints on the parameters μ_4 and μ_2 , it is known that the Schrodinger equation with the sextic anharmonic oscillator potential $V(r) = r^6 + \mu_4 r^4 + \mu_2 r^2$ is exactly solvable. Here, we solve the Schrodinger equation for arbitrary values of the potential parameters in the d -dimensional case. The method discussed offers a practical solution to the biconfluent heun equation's eigenvalue problem. A technique based on the Differential Galois Theory approach as well Asymptotic iteration method is used to compute explicit solutions and to evaluate the coefficients of the series solutions for arbitrary μ_4 and μ_2 . This is a joint work with Mourad E. Ismail and Nasser Saad.

Chain sequences and zeros of a perturbed R_{II} type recurrence relation

Swaminathan Anbhu

Indian Institute of Technology, Roorkee, India

In this talk, new algebraic and analytic aspects of the orthogonal polynomials satisfying R_{II} type recurrence relation given by

$$\mathcal{P}_{n+1}(x) = (x - c_n)\mathcal{P}_n(x) - \lambda_n(x - a_n)(x - b_n)\mathcal{P}_{n-1}(x), \quad n \geq 0,$$

where λ_n is a positive chain sequence and a_n, b_n, c_n are sequences of real or complex numbers with $\mathcal{P}_{-1}(x) = 0$ and $\mathcal{P}_0(x) = 1$ are investigated when the recurrence coefficients are perturbed. Specifically, representation of new perturbed polynomials (co-polynomials of R_{II} type) in terms of original ones with the interlacing and monotonicity properties of zeros are given. For finite perturbations, a transfer matrix approach is used to obtain new structural relations. Effect of co-dilation in the corresponding chain sequences and their consequences onto the unit circle are analysed. A particular perturbation in the corresponding chain sequence called complementary chain sequences and its effect on the corresponding Verblunsky coefficients is also studied.

This is joint work with V. Shukla.

Chebyshev polynomials and compositions

George E. Andrews

Pennsylvania State University, USA

The Theory of Compositions of integers has mostly been relegated to the very basic aspects of combinatorics. The object of this paper is to reveal their close relation to the Chebyshev polynomial $T_n(x)$ and $U_n(x)$. As a result, interesting combinatorial questions arise for compositions that have not been examined previously.

On Infinitely many rational approximants to $\zeta(3)$

Jorge Arvesú

Universidad Carlos III de Madrid, Spain

A set of Hermite–Padé approximation problems near infinity along with some extra conditions for the involved vector polynomial solutions is considered. These solutions constitute the numerator and denominator sequences of rational approximants to $\zeta(3)$. An asymptotic estimation for the error-approximation by using steepest descent method is discussed. Indeed, the new constructed rational approximants reprove the irrationality of $\zeta(3)$. A comparison of these rational approximants and Apéry’s approximants is given.

Padé approximants for functions with four branch points

Ahmad Barhoumi

University of Michigan, USA

This talk centers around the large-degree behavior of non-Hermitian orthogonal polynomials arising from Padé approximants (or “best” rational approximants). In particular, I will discuss approximants corresponding to a class of functions with four branch points. While it might seem modest, four branch points are already enough to generate certain singular, symmetric, arrangements where the orthogonal polynomials behave unexpectedly. I will highlight how these difficulties appear in the analysis and how one might remedy them. This talk is based on joint work with Maxim Yattselev.

Symmetric orthogonal polynomials

Peter Clarkson

University of Kent, UK

In this talk I will discuss symmetric orthogonal polynomials on the real line. Symmetric polynomials give rise to orthogonal systems which have important applications in spectral methods, with several important advantages if their differentiation matrix is skew-symmetric and highly structured. The symmetric orthogonal polynomials discussed will include a modified Hermite weight, which arises in random matrix theory, and generalised Freud weights, which arise in the matrix model in two-dimensional quantum gravity. It is well-known that orthogonal polynomials satisfy a three-term recurrence relation. I will show that for modified Hermite weight the coefficients in the recurrence relation are expressed in terms of special function solutions of the fifth Painlevé equation and for the generalised Freud weights in terms of solutions of Wronskians of generalised hypergeometric functions.

The antisymmetry relation for continuous q -Jacobi polynomials

Howard Cohl

NIST, USA

By comparing Gauss hypergeometric representations of the antisymmetric Jacobi polynomials and the Gegenbauer function of the first kind one may obtain a proportional relation between these two objects. The Gegenbauer function of the first kind is the standard continuation of the Gegenbauer (ultraspherical) polynomials when the degree of the polynomial is a complex number. In order to obtain a q -analogue of this relation, we use Rahman's Askey–Wilson function of the first kind which is the continuation of the Askey–Wilson polynomial when its degree is allowed to be complex. We specialize the Rahman's Askey–Wilson function of the first kind to obtain two types of continuous q -Jacobi functions of the first kind and corresponding continuous q -ultraspherical/Rogers functions of the first kind. Using these functions we derive q -analogues of the antisymmetry relation for continuous q -Jacobi polynomials.

Quantum states in random environments

David Damanik

Rice University, USA

In this talk we discuss some recent work on quantum states in random environments in the absence of independence. After recalling some of the key results in the case where independence holds (i.e., for the Anderson model), we explain the difficulties in establishing extensions of them when correlations are allowed and how they are overcome in one space dimension with the help of dynamical systems methods.

A new partition statistic

Madeline Dawsey

University of Texas - Tyler, USA

Partition theory has traditionally revolved around the size statistic for partitions and the partition function $p(n)$, which counts the number of partitions of size n . In recent work with Just, Schneider, and Sharp, I have explored a new perspective on partitions that focuses more on multiplicative aspects of partitions than their classical additive structure. In this talk, I will give a short overview of partitions from an additive standpoint and then define a new, multiplicative partition statistic called the “supernorm”. I will also present several different applications of the supernorm in combinatorics, analytic number theory, and algebraic number theory.

Szegő condition, scattering, and vibration of Krein strings

Sergey Denisov

University of Wisconsin-Madison, USA

I will discuss the recent work with R. Bessonov in which we obtained the dynamical characterization of measures on the real line that have convergent logarithmic integral. That is done in the framework of de Branges canonical systems. When applied to the Dirac operator and Krein’s equation of vibrating string, our results yield the criteria for “scattering” both in terms of spectral data and in terms of coefficients.

Discrete Krall polynomials

Manuel Dominguez de la Iglesia

Universidad Nacional Autonoma de Mexico

One of the main contributions of L. Littlejohn in his career is the study of Krall polynomials, i.e. orthogonal polynomials satisfying higher-order differential equations. In this talk I will give a review of how to construct discrete Krall polynomials (satisfying now higher-order difference equations) related to the classical families of Charlier, Meixner, Krawtchouk and Hahn polynomials. The content of this talk is based on several joint works with A.J. Durán.

Zeros of Jacobi polynomials

Kathy Driver

University of Cape Town, South Africa

Suppose $\{P_n^{(\alpha,\beta)}(x)\}_{n=0}^\infty$ is a sequence of Jacobi polynomials, $\alpha, \beta > -1$. It is known that the zeros of $P_n^{(\alpha,\beta)}(x)$ and $P_n^{(\alpha-t,\beta+s)}(x)$ are interlacing for $\alpha - t > -1, \beta > -1, 0 \leq t, s \leq 2$. We discuss the simplest cases of a question raised by Alan Sokal at OPSFA 2019 whether the zeros of $P_n^{(\alpha,\beta)}(x)$ and $P_{n+k}^{(\alpha+t,\beta+s)}(x)$ are interlacing when $s, t > 0$ and $k, n \in \mathbb{N}$. We prove that the zeros of $P_n^{(\alpha,\beta)}(x)$ and $P_{n+1}^{(\alpha,\beta+1)}(x)$, $\alpha > -1, \beta > 0, n \in \mathbb{N}$ are partially, but in general not fully, interlacing, depending on the values of α, β and n . We also consider interlacing of the zeros of $P_n^{(\alpha,\beta)}(x)$ and $P_{n+1}^{(\alpha+1,\beta+1)}(x)$, $\alpha, \beta > -1$ and provide examples confirming that our results cannot be strengthened in general. This is joint work with Jorge Arvesú Carballo and Lance Littlejohn.

Exceptional Hahn and Jacobi polynomials with an arbitrary number of continuous parameters

Antonio Durán

Universidad de Sevilla, Spain

Exceptional polynomials are orthogonal polynomials with respect to a measure which are also eigenfunctions of a second order difference or differential operator. In mathematical physics, they allow the explicit computation of bound states of rational extensions of classical quantum-mechanical potentials. The most apparent difference between classical or classical discrete orthogonal polynomials and their exceptional counterparts is that the exceptional families have gaps in their degrees, in the sense that not all degrees are present in the sequence of polynomials. In this talk, we introduce new examples of exceptional Hahn and Jacobi polynomials. The new examples have the novelty that they depend on an arbitrary number of continuous parameters. These families are constructed by dualizing Krall dual Hahn polynomials. Krall polynomials are orthogonal polynomials which are eigenfunctions of a higher order differential or difference operator. The Krall dual Hahn families provide further examples for the problem explicitly posed by Richard Askey in 1991.

An approach to universality using Weyl m -functions

Benjamin Eichinger

Vienna University of Technology, Austria

We describe an approach to universality limits for orthogonal polynomials on the real line which is completely local and uses only the boundary behavior of the Weyl m -function at the point. We show that bulk universality of the Christoffel-Darboux kernel holds for any point where the imaginary part of the m -function has a positive finite nontangential limit. This approach is based on studying a matrix version of the Christoffel-Darboux kernel and the realization that bulk universality for this kernel at a point is equivalent to the fact that the corresponding m -function has normal limits at the same point. Our approach automatically applies to other self-adjoint systems with 2×2 transfer matrices such as continuum Schrödinger and Dirac operators. We also obtain analogous results for orthogonal polynomials on the unit circle.

The talk is based on a joint work with Milivoje Lukić and Brian Simanek.

A Sobolev multi-orthogonal polynomial's approach to bulk queueing theory

Ulises Fidalgo

Case Western Reserve University, USA

We consider a stationary Markov process that models certain queues with a bulk service of a fixed number m of admitted customers. We find an integral expression of its transition probability function in terms of certain Sobolev multi-orthogonal polynomials. These integral's paths are starlike subsets of the complex plane.

Spectral properties of the unitary almost-Mathieu operator

Jake Fillman

Texas State University, USA

We introduce a unitary almost-Mathieu operator, which is a one-dimensional quasi-periodic quantum walk obtained from an anisotropic two-dimensional quantum walk in a uniform magnetic field. We will discuss background information, the origins of the model, its interesting spectral features, and key ideas needed in proofs of the main results. [Joint work with Christopher Cedzich, Darren C. Ong, and Zhenghe Zhang]

Multi-scale analysis for the random XXZ higher spin chain

Lee Fisher

University of California - Irvine, USA

We study the Heisenberg XXZ chain with local spin J with a random background magnetic field. As in the spin-1/2 case, it's possible to rewrite the Hamiltonian as a direct sum of N -particle Schrodinger operators with attractive interaction. We will focus on the multiscale analysis approach to prove several localization results for this system.

A spectral exploration of exceptional Laguerre operators

Jessica S. Kelly

Christopher Newport University, USA

In this talk, we will apply the theory of boundary triples to the exceptional Laguerre polynomials in order to obtain the Weyl m -function for self-adjoint extensions. The formulation of boundary triples in the exceptional case relies on the close relationship between the classical and exceptional orthogonal polynomial families. In particular, we exploit the intertwining properties of the differential expressions and the fact that the exceptional orthogonal polynomial solutions may be obtained by applying a Darboux transformation to the classical orthogonal polynomials.

Additionally, we consider more general Laguerre-type exceptional orthogonal families formed via a general sequence of Darboux transformations. Here the formulation of exceptional Laguerre polynomials through partitions is utilized. Changes to the spectral and Laguerre-specific parameters are followed throughout the process; tracing these changes from the classical to exceptional case allows for characterization of spectral properties for general Laguerre-type operators.

This is joint work with Dale Frymark.

Finite-rank perturbations and applications

Constanze Liaw

University of Delaware and NSF, USA

Finite-rank perturbations arise naturally, for example, when several boundary conditions of a differential operator are changed simultaneously. Through their connection with analytic function theory, they have been of interest to the community for some time. Recent advances have shown that (and at times how) their theory is more involved than that of rank one perturbations. We will highlight some key results, as well as some applications. In particular, we will discuss results for powers of the derivative and sketch how merging of boundary triples with finite-rank perturbation theory provides a more holistic view of an operator's spectral information.

A Survey of left-definite operator theory with applications to orthogonal polynomials

Lance L. Littlejohn
Baylor University, USA

If A is a self-adjoint operator that is bounded below in a Hilbert space H , Littlejohn and Wellman in 2002 show that this operator generates a certain continua of unique Hilbert (scales) spaces $\{H_r\}_{r>0}$ and unique self-adjoint operators $\{A_r\}_{r>0}$. For reasons originating in the theory of differential operators, H_r is called the r^{th} left-definite space and A_r the r^{th} left-definite operator associated with (H, A) .

The concept of left-definite theory can be traced back to early work of Hermann Weyl and later developed in more detail by F. W. Schäfke and A. Schneider in Germany and by Å. Pleijel in Sweden in the 1960's. In this lecture, we give a brief overview of this theory and apply it to several examples including the classical second-order differential operators having the classical orthogonal polynomials of Jacobi (including Legendre), Laguerre, and Hermite as eigenfunctions. Another example that we consider in this talk is a non-classical Laguerre case, namely when the Laguerre parameter α is a negative integer. In this case, the Laguerre polynomials $\{L_n^\alpha\}_{n=0}^\infty$ are orthogonal in some Sobolev space W_α ; we apply the left-definite theory to construct a self-adjoint operator A_α , generated by the second-order Laguerre differential expression, in W_α .

One of the interesting side consequences of the Jacobi example is the connection to new(ish) combinatorial numbers which generalize the classical Stirling numbers of the second kind (which, incidently, appear in the left-definite analysis of both the Laguerre and Hermite examples).

Fermi isospectrality for discrete periodic Schrödinger operators

Wencai Liu
Texas A&M University, USA

Let $\Delta + V$ be the discrete Schrödinger operator, where Δ is the discrete Laplacian on \mathbb{Z}^d and the potential $V : \mathbb{Z}^d \rightarrow \mathbb{R}$ is Γ -periodic. We prove two rigidity theorems for discrete periodic Schrödinger operators in any dimension $d \geq 3$: 1) if V and Y are Fermi isospectral (that is, at some energy level, Fermi varieties of the Γ -periodic potential V and the Γ -periodic potential Y are the same), and Y is a separable function, then V is separable as well; 2) if potentials V and Y are Fermi isospectral and both $V = \bigoplus_{j=1}^r V_j$ and $Y = \bigoplus_{j=1}^r Y_j$ are separable functions, then, up to a constant, lower dimensional decompositions V_j and Y_j are Floquet isospectral, $j = 1, 2, \dots, r$.

Lattice paths, vector continued fractions, and resolvents of banded Hessenberg operators

Abey López-García

University of Central Florida, USA

In this talk I will discuss a combinatorial interpretation of vector continued fractions that are obtained by applying the Jacobi–Perron algorithm to a vector of Weyl (resolvent) functions of a banded Hessenberg operators. The interpretation consists in identifying the coefficients in the Laurent series expansion of the Weyl functions as “labeled polynomials” of certain families of lattice paths. In the scalar case this interpretation reduces to the well-known relation between Jacobi continued fractions and Motzkin paths. I will discuss the particular case of bi-diagonal Hessenberg operators, which generate the generalized Stieltjes–Rogers polynomials, and are associated with the generalized Dyck paths. This talk is based on a joint work with Vasilij A. Prokhorov (U. South Alabama).

Fixed point theorems and strong asymptotics of multi-level Hermite–Padé polynomials

Guillermo López-Lagomasino

Universidad Carlos III de Madrid, Spain

We give a Szegő type formula which describes the strong asymptotic of the polynomials which arise in a mixed type Hermite–Padé approximation problem of a Nikishin system of functions.

Coherent pairs of measures on the unit circle

Francisco Marcellán

Universidad Carlos III de Madrid, Spain

In this talk we deal with pairs of measures supported on the unit circle such that the corresponding sequences of orthogonal polynomials are related by a differential/difference relation. We describe some examples of such measures. An application to the analysis of the polynomials orthogonal with respect to Sobolev inner product associated with such a pair of measures This is a joint work with A. Sri Ranga (UNESP, Brazil).

Localization and eigenvalue statistics within Hartree–Fock theory

Rodrigo Matos
Texas A&M University, USA

I will discuss recent results related to Anderson localization for discrete random operators within the Hartree–Fock approximation. After introducing the model, which incorporates interactions among particles through a mean field, I will present localization results based on joint work with Schenker. Time allowing, eigenvalue statistics in the localization regime for these models will also be discussed.

On another characterization of Askey–Wilson polynomials

Dieudonne Mbouna
University of Almería, Spain

In this talk we expose the theory of classical orthogonal polynomials on lattices and we use this to give a Al-Salam and Chihara type characterization of classical orthogonal polynomials on lattices.

On the convergence of Hermite–Padé approximants

Sergio Medina-Peralta
Florida International University, USA

We study the convergence of sequences Hermite–Padé approximants for certain systems of meromorphic functions made up of perturbations of Nikishin systems of functions. We will present an extension of Markov’s theorem as well as some relative asymptotic results.

Optimality of the Birman–Hardy–Rellich inequalities

Isaac Michael
Louisiana State University, USA

In 1961, Birman proved a sequence of inequalities valid for functions in $C_0^n((0, \infty))$ containing the classical (integral) Hardy inequality and the well-known Rellich inequality, and much effort has been made in improving and extending these inequalities with weights and singular logarithmic refinement terms.

In this talk, we discuss the optimality of the Birman inequalities. In particular, we introduce a new proof of the power-weighted Birman inequalities without refinement terms, using a modified variable transformation in integrals first studied by Hartman and Müller-Pfeiffer.

This is based on recently published work with Fritz Gesztesy, Lance Littlejohn, and Michael Pang.

The spectral theory of exceptional Hermite polynomials

Robert Milson

Dalhousie University, Canada

In my talk I would like to revisit exceptional Hermite polynomials from the point of view of spectral theory, following the work initiated by Lance Littlejohn. Adapting a result of Deift, we provide an alternative proof of the completeness of these polynomial families. In addition, using equivalence of Hermite Wronskians we characterize the possible gap sets for the class of exceptional Hermite polynomials.

AGM and jellyfish swarms of elliptic curves

Ken Ono

University of Virginia, USA

The classical AGM produces wonderful infinite sequences of arithmetic and geometric means with common limit. This theory relies critically on the ${}_2F_1$ hypergeometric functions of Gauss. For finite fields $GF(q)$, with $q \equiv 3 \pmod{4}$, we introduce a finite field analogue AGMF $_q$ that spawns directed finite graphs instead of infinite sequences. The compilation of these graphs reminds one of a jellyfish swarm, as the 3D renderings of the connected components resemble jellyfish (i.e., tentacles connected to a bell head). These swarms turn out to be more than the stuff of child's play; they are taxonomical devices in number theory. Each jellyfish is an isogeny graph of elliptic curves with isomorphic groups of $GF(q)$ -points. We will describe this theory in this lecture, whose players include Gaussian hypergeometric functions and Gauss' class numbers of binary quadratic forms.

Weighted Birman–Hardy–Rellich inequalities with logarithmic refinements

Michael Pang

University of Missouri-Columbia, USA

We extend Birman's sequence of integral inequalities originally obtained in 1961, and containing Hardy's and Rellich's inequality as special cases, to a sequence of inequalities that incorporates power weights x^α for x varying in interval $(0, \rho)$, $\rho \in (0, \infty) \cup \{\infty\}$, on either side and logarithmic refinements on the smaller side of the inequality as well. Employing a new technique of proof relying on a combination of transforms originally due to Hartman (1948) and Müller-Pfeiffer (1981), the parameter $\alpha \in \mathbb{R}$ in the power weights is now unrestricted, considerably improving on prior results in the literature. This continues a tradition of logarithmic refinements in connection with Hardy's inequality, going back to work in oscillation theory by A. Kneser (1893), Hartman (1948), Hille (1948), and Rellich (1951), resulting in a sequence of sharp statements of boundedness from below by zero of a class of homogeneous $2m$ -th order differential operators on $C_0^\infty((0, \rho))$. We also discuss extensions of these inequalities to multi-dimensions. (Joint work with Fritz Gesztesy, Lance L. Littlejohn, and Isaac Michael.)

Determinantal formulas for exceptional orthogonal polynomials

Brian Simanek

Baylor University, USA

We will show how to express exceptional orthogonal polynomials as the determinants of special matrices. The entries of these matrices are powers of zeros of classical orthogonal polynomials, except in the last row, where the entries are polynomials with concise formulas. Our most general results apply to the exceptional Jacobi and exceptional Laguerre polynomials, though we can say something about exceptional Hermite polynomials as well.

A tale of three coauthors: comparison of Ising models

Barry Simon

California Institute of Technology, USA

On Friday, Jan 14, I had a draft of a single author paper intended for the Lieb Festschrift. Six days later, the paper had three coauthors. This talk will explain the interesting story, expose some underlying machinery and sketch the proof of a lovely inequality on certain finite sums.

Solvability of some integro-differential equations with drift and superdiffusion

Vitali Vougalter

University of Toronto, Canada

We establish the existence in the sense of sequences of solutions for some integro-differential type equations containing the drift term and the square root of the one dimensional negative Laplacian, on the whole real line or on a finite interval with periodic boundary conditions in the corresponding H^2 spaces. The argument relies on the fixed point technique when the elliptic equations involve first order differential operators with and without the Fredholm property. It is proven that, under the reasonable technical assumptions, the convergence in L^1 of the integral kernels implies the existence and convergence in H^2 of solutions.

Recent progress on exact moments of entanglement entropies

Lu Wei

Texas Tech, USA

We discuss recent results on the exact moments of entanglement entropies (including von Neumann entropy and quantum purity) over different models of generic quantum states. Main ingredients of the computations are classical and modern orthogonal polynomial systems and the observed anomaly cancellation phenomena.

Gap probabilities for the Bures–Hall ensemble and the Cauchy–Laguerre two-matrix model

Nicholas Witte

Texas Tech, USA

The Bures metric and the associated Bures–Hall measure is arguably the best choice for studying the spectrum of the quantum mechanical density matrix with no a priori knowledge about the system. We investigate the probability of a gap in the spectrum of this model, either at the bottom $[0, s)$ or at the top $(s, 1]$, utilising the connection of this Pfaffian point-process with the allied problem in the determinantal point-process of the Cauchy–Laguerre bi-orthogonal polynomial system. To this end we develop new general results about Cauchy bi-orthogonal polynomial system for a more general class of weights: in particular a new Christoffel–Darboux formula, reproducing kernels and differential equations for the polynomials and their associated functions. This two-dimensional system is most simply expressed with rank-3 matrix variables and possesses an associated bilinear form with metric. Furthermore under specialisation to Laguerre type densities in the weight we construct the differential equations in two deformation variables, and observe that the recurrence, spectral and deformation derivative structures form a compatible and integrable triplet of Lax equations.

The complex zeros of random orthogonal polynomials

Aaron Yeager

College of Coastal Georgia, USA

We utilize Cauchy’s argument principle in combination with the Jacobian of a holomorphic function in several complex variables and the first moment of a ratio of two correlated complex normal random variables to prove explicit formulas for the density and the mean distribution of complex zeros of random polynomials spanned by orthogonal polynomials on the unit circle and on the unit disk. We then inquire into the consequences of their asymptotical evaluations.

Self-adjoint ordinary differential operators

Anton Zettl

A survey of the theory of self-adjoint ordinary differential operators in Hilbert space. The recently developed theory of symmetric expressions and boundary conditions which determine these operators. A review of the recent solution of the deficiency index problem.

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