# Invited Plenary Speakers

## Donatella Danielli

`Title:` Regularity properties in obstacle-type problems for higher-order fractional powers of the Laplacian

`Abstract:` In this talk we will discuss a sampler of obstacle-type problems associated with the fractional Laplacian $(-\Delta)^s$, for $1 < s < 2$. Our goals are to establish regularity properties of the solution and to describe the structure of the free boundary. To this end, we combine classical techniques from potential theory and the calculus of variations with more modern methods, such as the localization of the operator and monotonicity formulas. The results have been obtained in joint work with Alaa Haj Ali (Arizona State University) and Arshak Petrosyan (Purdue University).

## Martin Dindos

`Title:` New progress on solvability of Regularity problem for elliptic operators with coefficients satisfying large Carleson condition

`Abstract:` Recall that when $Lu = div(A\nabla u)$ and coefficients $A$ satisfy Carleson condition (that is $|\nabla A|^2\delta$ is a Carleson measure) then the corresponding elliptic measure is $A_\infty$ and hence the $L^p$ Dirichlet problem is solvable for some large $p < \infty$. There is also an analogous smallness result, namely that on $C^1$ domains $L^p$ Dirichelt problem is solvable for all $1 < p < \infty$, provided $|\nabla A|^2\delta$ is a vanishing Carleson measure.

In the case of Regularity problem, the analogous smallness result was established in Dindos-Pipher-Rule. We have recently (jointly with S. Hofmann and J. Pipher) established an $n$-dimensional reduction that relates solvability of Regularity problem to the solvability of Regularity problem for a block-form operator. This has allowed us to fully resolve the question of solvability of Regularity and Neumann problem for $1 < p < 1 + \epsilon$ in dimension 2 when $|\nabla A|^2\delta$ is a LARGE Carleson measure. The case $n > 2$ remains open, but is reduced to the case of a block-form matrix.

## Jameson Graber

`Title:` The Master Equation in Mean Field Games

`Abstract:` Mean field games model strategic interactions among a large number of agents. By assuming all the agents are essentially interchangeable, one can reduce the complexity of the system by focusing on the interaction between a "representative agent," who solves an optimal control problem, and the surrounding population, which is modeled by a probability distribution. P.-L. Lions has introduced a new PDE, called the Master Equation, whose solution is supposed to be the value function for the representative agent's optimal control problem, depending on the initial distribution

of the population. We thus have a PDE in which one of the independent variables is a probability measure. This calls for a new differential calculus and new techniques to prove existence, uniqueness, and regularity of solutions. In this presentation, I will give a heuristic derivation of the Master Equation, introduce some notions of derivatives on the space of probability measures, and discuss some recent progress toward solving the Master Equation.

**Piotr Hajłasz**

`Title:` Approximation of mappings with derivatives of low rank

`Abstract:` My talk is based on two recent joint papers with Paweł Goldstein.

Jacek Gałęski in 2017, in the context of his research in geometric measure theory, formulated the following conjecture:

**Conjecture.** Let $1 \le m < n$ be integers and let $\Omega \subset \mathbb{R}^n$ be open. If $f \in C^1(\Omega, \mathbb{R}^n)$ satisfies $\operatorname{rank} Df \le m$ everywhere in $\Omega$, then $f$ can be uniformly approximated by smooth mappings $g \in C^\infty(\Omega, \mathbb{R}^n)$ such that $\operatorname{rank} Dg \le m$ everywhere in $\Omega$.

One can also modify the conjecture and ask about a local approximation: smooth approximation in a neighborhood of any point. These are very natural problems with possible applications to PDEs and Calculus of Variations. However, the problems are difficult, because standard approximation techniques like the one based on convolution do not preserve the rank of the derivative. It is a highly nonlinear constraint, difficult to deal with.

In 2018 Goldstein and Hajłasz obtained infinitely many counterexamples to this conjecture. Here is one:

**Example.** There is $f \in C^1(\mathbb{R}^5, \mathbb{R}^5)$ with $\operatorname{rank} Df \le 3$ that cannot be locally and uniformly approximated by mappings $g \in C^2(\mathbb{R}^5, \mathbb{R}^5)$ satisfying $\operatorname{rank} Dg \le 3$.

This example is a special case of a much more general result and the construction heavily depends on algebraic topology including the homotopy groups of spheres and the Freudenthal suspension theorem.

More recently Goldstein and Hajłasz proved the conjecture in the positive in the case when $m = 1$. The proof is based this time on methods of analysis on metric spaces and in particular on factorization of a mapping through metric trees.

The method of factorization through metric trees used in the proof of the conjecture when $m = 1$ is very different and completely unrelated to the methods of algebraic topology used in the construction of counterexamples. However, quite surprisingly, both techniques have originally been used by Wenger and Young as tools for study of Lipschitz homotopy groups of the Heisenberg group, a problem that seems completely unrelated to problems discussed in this talk.

**Steve Hofmann**

`Title:` Quantitative absolute continuity of caloric measure
`Abstract:` For an open set $\Omega \subset \mathbb{R}^d$ with an Ahlfors regular boundary, solvability of the Dirichlet problem for Laplace's equation, with boundary data in $L^p$ for some $p < 1$, is equivalent to quantitative, scale invariant absolute continuity (more precisely, the weak-$A_\infty$ property) of harmonic measure with respect to surface measure on $\partial\Omega$. A similar statement is true in the caloric setting. Thus, it is of interest to find geometric criteria which characterize the open sets for which such absolute continuity (hence also solvability) holds. Recently, this has been done in the harmonic case, but in the caloric setting, matters are still at a more primitive stage. On the other hand, we now have a definitive characterization at least in the case that the boundary is known in advance to be given (locally) as a Lip(1,1/2) graph, and in turn, this leads to a result of free boundary type in which, more generally, quantitative absolute continuity of caloric measure, with respect to "surface measure" on the parabolic Ahlfors regular (lateral) boundary $\Sigma$, implies parabolic uniform rectifiability of $\Sigma$.

This is joint work with S. Bortz, J. M. Martell and K. Nyström.

**Tao Mei**

`Title:` A Marcinkiewicz-Littlewood-Paley Multiplier theory for Operators
`Abstract:` The Schur product of two n by n matrices is a pointwise product. More precisely, the Schur product of $m = (m_{ij})$ and $A = (a_{ij})$ is the matrix $(m_{ij}a_{ij})$. Let us fix the matrix m and view the Schur product of matrices with m as a map from matrices to matrices. We call this map a Schur multiplier. The Schur multipliers on matrices (for n finite or infinite) have a natural connection to Fourier multiplier on Euclidean spaces. It is well known that, the complete operator bounds of Toeplitz-type Schur multipliers is the same to that of the corresponding Fourier multipliers on matrix-valued functions. In this talk, I plan to explain how such connections extend to general Schur multipliers and introduce an analogue of Marcinkiewicz-multiplier theory for Schur multipliers on Schatten-p classes. If time permits, I will also explain a related work by J. Parcet etc. The talk will be based on a recent work with ChianYeong Chuah and Zhenchuan Liu.

**Michael Taylor**

`Title:` Euler equation on a rotating surface
`Abstract:` We discuss the 2D Euler equation for ideal incompressible fluid flow on a rotating surface, subject to the effect of the Coriolis force, with an emphasis on surfaces of revolution (though not necessarily the standard sphere). We bring in conservation laws that yield global existence of solutions, with a double exponential

estimate. We examine families of stationary solutions, and discuss the question of stability, bringing in a variant of the Arn'old method, and noting how the rotation speed can act to enhance stability of zonal fields. We mention evidence from numerical study of the equation of linear stability that there are mysteries to be resolved regarding just how this dependence works. This latter part involves joint work with Jeremy Marzuola.

**Magdalena Toda**

`Title:` Elastic Energies in Geometric Analysis and Biological Applications
`Abstract:` The talk presents the theory and applications of functionals involving surface curvature, which we refer to as elastic energies or generalized Willmore energies. Biological applications include protein folding, red blood cells and biomembranes. The speaker and coauthors present their recently published and unpublished results on critical points of these energies, stability, and related generalized Willmore flows. Cases of generalized Willmore surfaces with fixed and free boundaries will also be presented, if time permits.

## Invited Short Talks

### Artur H. O. Andrade

`Title:` Fredholm theory for boundary value problems associated with even powers of the Laplacian
`Abstract:` In this talk I will discuss the Dirichlet problem for even powers of the Laplacian in regular SKT domains in $\mathbb{R}^n$. In connection with this I will analyze a family of bilinear forms associated with even powers of the Laplacian and the associated double multi-layer potential operators arising in this setting.

### Pedro T. F. da Silva

`Title:` Beurling-Hardy Spaces and The Neumann Problem for the Laplacian
`Abstract:` We introduce the class Beurling-Hardy spaces in the setting of Ahlfors-David regular sets, develop a Calderón-Zygmund theory for singular integral operators acting on this scale of spaces, and discuss the solvability of the Neumann problem for the Laplacian, with boundary data in Beurling-Hardy spaces, via the method of boundary layer potentials. This is joint work with Marius Mitrea.

### Joengsu Kyeong

`Title:` Singular integral operators associated with second order elliptic systems in two dimensions `Abstract:` The goal of this talk is to investigate coefficient tensors associated with second order elliptic operators in infinite sectors in $\mathbb{R}^2$ and properties of the corresponding singular integral operators, using Mellin transform techniques. Specifically the discussion is focused on the relationship between distinguished coefficient tensors (those leading to chord-dot-normal type double layer potentials) and $L^p$ spectral and Hardy kernel properties of the associated singular integral operators.

### Marcus Laurel

`Title: Boundary value problems in weighted Morrey spaces`
`Abstract:` The goal of this talk is to present solvability results for boundary value problems for elliptic systems in the class of $\delta$-AR domains (where $\delta > 0$ is sufficiently small), with boundary data in Muckenhoupt weighted Morrey spaces. Our approach relies on boundary layer potentials, and we develop a comprehensive Calderón-Zygmund theory for singular integral operators on Morrey spaces and their preduals (a.k.a. block spaces).